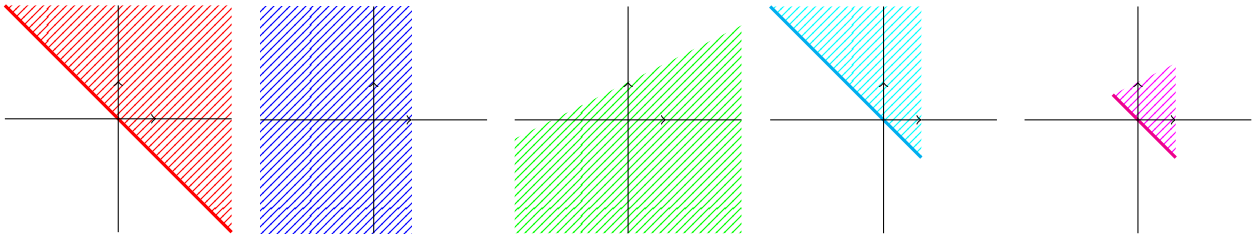
Fiche 16 : FONCTIONS DE DEUX VARIABLES – CORRIGÉ

Exercice 1. On obtient les représentations graphiques suivantes :

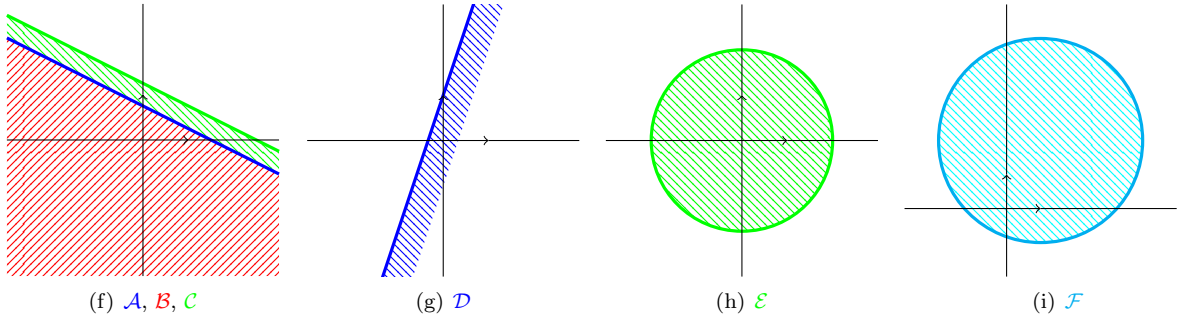
$$P_1 = \{(x,y) \in \mathbb{R}^2 / \, x+y \geq 0\} \qquad P_2 = \{(x,y) \in \mathbb{R}^2 / \, x < 1\} \qquad P_3 = \{(x,y) \in \mathbb{R}^2 / \, x-2y+2 > 0\}$$



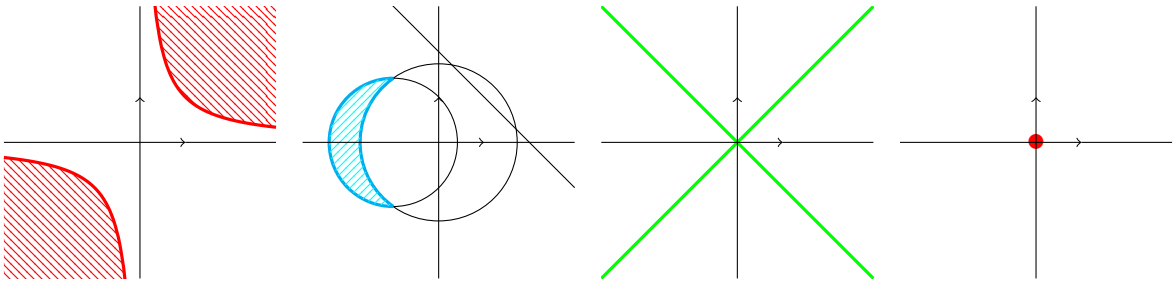
(a) P_1 (b) P_2 (c) P_3 (d) $P_1 \cap P_2$ (e) $P_1 \cap P_2 \cap P_3$

Exercice 2. On obtient les représentations graphiques suivantes :

$$\mathcal{A} = \{(x,y) \in \mathbb{R}^2 \mid 2x+4y = 3\} \qquad \mathcal{B} = \{(x,y) \in \mathbb{R}^2 \mid 2x+4y < 3\}$$
$$\mathcal{C} = \{(x,y) \in \mathbb{R}^2 \mid 3 < 2x+4y \leqslant 5\} \qquad \mathcal{D} = \{(x,y) \in \mathbb{R}^2 \mid -1 < -3x+y \leqslant 1\}$$
$$\mathcal{E} = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leqslant 4\} \qquad \mathcal{F} = \{(x,y) \in \mathbb{R}^2 \mid (x-1)^2+(y-2)^2 \leq 9\}$$
$$\mathcal{G} = \{(x,y) \in \mathbb{R}^2 \mid xy \geqslant 1\} \qquad \mathcal{H} = \{(x,y) \in \mathbb{R}^2 \mid (x+1)^2+y^2 \leqslant 2,\ x^2+y^2 \geqslant 3,\ x+y \leqslant 2\}$$
$$\mathcal{I} = \{(x,y) \in \mathbb{R}^2 \mid x^2-y^2 = 0\} \qquad \mathcal{J} = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 = 0\}.$$



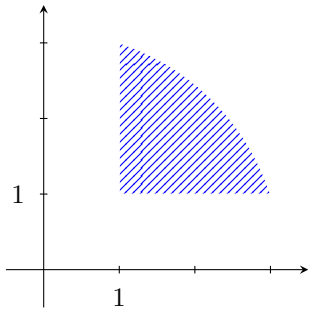
(f) $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ (g) $\mathcal{D}$ (h) $\mathcal{E}$ (i) $\mathcal{F}$



(j) $\mathcal{G}$ (k) $\mathcal{H}$ (l) $\mathcal{I}$ (m) $\mathcal{J}$

Exercice 3. Si l'on considère que le bord courbe est un arc de cercle centré en l'origine, son rayon vaut $\sqrt{10}$. On peut aussi considérer qu'il s'agit d'un arc de cercle centré en $(1,1)$, et de rayon 2.
On obtient alors deux représentations différentes pour $\mathcal{K}$:

$$\begin{aligned}\mathcal{K} &= \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 < 10,\ x>1,\ y>1\}\\ &= \{(x,y) \in \mathbb{R}^2 \mid (x-1)^2+(y-1)^2 < 4,\ x>1,\ y>1\}\end{aligned}$$



Exercice 4.

fonction	$\mathcal{D}$	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial^2}{\partial x^2}$	$\frac{\partial^2}{\partial y^2}$	$\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$
$a(x,y)$	$\mathbb{R}^2$	$4x^3+3x^2y+2xy^2+y^3$	$x^3+2x^2y+3xy^2+4y^3$	$12x^2+6xy+2y^2$	$2x^2+6xy+12y^2$	$3x^2+4xy+3y^2$
$\ln(x^2+y^2)$	$\mathbb{R}^2 \setminus \{(0,0)\}$	$\frac{2x}{x^2+y^2}$	$\frac{2y}{x^2+y^2}$	$-2\frac{x^2-y^2}{\left(x^2+y^2\right)^2}$	$2\frac{x^2-y^2}{\left(x^2+y^2\right)^2}$	$\frac{-4xy}{\left(x^2+y^2\right)^2}$
$(x^2+y^2)e^{-xy}$	$\mathbb{R}^2$	$\frac{2x-x^2y-y^3}{e^{-xy}}$	$\frac{2y-x^3-xy^2}{e^{-xy}}$	$\frac{-4xy+x^2y^2+y^4+2}{e^{-xy}}$	$\frac{-4xy+x^4+x^2y^2+2}{e^{-xy}}$	$\frac{-3y^2+x^3y+xy^3-3x^2}{e^{-xy}}$
$x(\ln y)^2+y^2$	$y>0$	$(\ln y)^2$	$2\frac{x\ln(y)+y^2}{y}$	0	$2\frac{x+y^2-x\ln(y)}{y^2}$	$2\frac{\ln(y)}{y}$
$(\ln y)^2+2\ln y+x^2$	$y>0$	$2x$	$2\frac{\ln(y)+1}{y}$	2	$-2\frac{\ln(y)}{y^2}$	0
$\frac{x-y}{x^2+y}$	$y>-x^2$	$-\frac{x^2-y-2xy}{\left(x^2+y\right)^2}$	$-\frac{x(x+1)}{\left(x^2+y\right)^2}$	$2\frac{x^3-3xy-3x^2y+y^2}{\left(x^2+y\right)^3}$	$\frac{2x(x+1)}{\left(x^2+y\right)^3}$	$\frac{2x^3-2xy+3x^2-y}{\left(x^2+y\right)^3}$