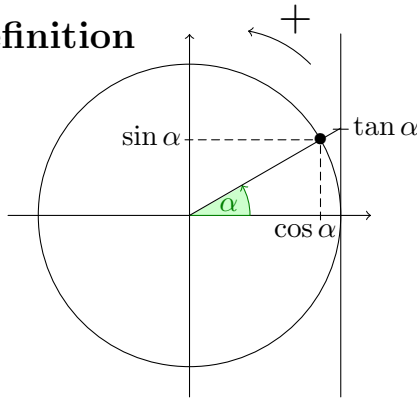


Formulaire de TRIGONOMÉTRIE

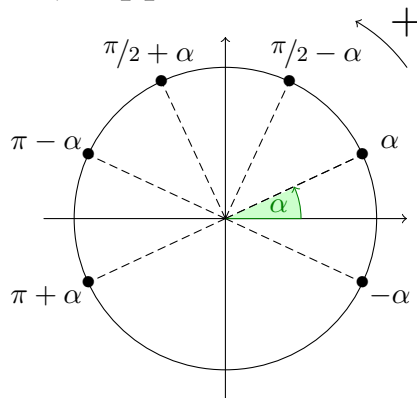
1 Définition



Pythagore : $\cos^2 \alpha + \sin^2 \alpha = 1$

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

2 Angles opposés, complémentaires, supplémentaires



- $\cos \alpha = \cos(-\alpha) = -\cos(\pi - \alpha) = -\cos(\pi + \alpha)$
 $= \sin\left(\frac{\pi}{2} - \alpha\right) = \sin\left(\frac{\pi}{2} + \alpha\right)$
- $\sin \alpha = \sin(\pi - \alpha) = -\sin(-\alpha) = -\sin(\pi + \alpha)$
 $= \cos\left(\frac{\pi}{2} - \alpha\right) = -\cos\left(\frac{\pi}{2} + \alpha\right)$
- $\tan \alpha = \tan(\pi + \alpha) = -\tan(-\alpha) = -\tan(\pi - \alpha)$

3 Angle moitié

Avec $t = \tan \alpha/2$:

$$\cos \alpha = \frac{1 - t^2}{1 + t^2} \quad \sin \alpha = \frac{2t}{1 + t^2} \quad \tan \alpha = \frac{2t}{1 - t^2}$$

4 Valeurs remarquables

α	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos \alpha$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
$\sin \alpha$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\tan \alpha$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	\times

5 Duplication / linéarisation

- $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
 $= 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$
- $\sin 2\alpha = 2 \cos \alpha \sin \alpha$
- $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$
- $\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$
- $\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$

6 Formules d'addition

$$\begin{aligned} \cos(a + b) &= \cos a \cos b - \sin a \sin b \\ \sin(a + b) &= \sin a \cos b + \cos a \sin b \\ \tan(a + b) &= \frac{\tan a + \tan b}{1 - \tan a \tan b} \\ \cos a + \cos b &= 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) \\ \cos a - \cos b &= -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\ \sin a + \sin b &= 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) \\ \sin a - \sin b &= 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\ a \cos x + b \sin x &= r \cos(x - \theta) \text{ où } a + ib = r e^{i\theta} \end{aligned}$$

7 Formules de multiplication

$$\begin{aligned} \cos a \cos b &= \frac{\cos(a - b) + \cos(a + b)}{2} \\ \sin a \sin b &= \frac{\cos(a - b) - \cos(a + b)}{2} \\ \sin a \cos b &= \frac{\sin(a - b) + \sin(a + b)}{2} \end{aligned}$$

8 Équations trigonométriques

$$\begin{aligned} \cos a = \cos b &\iff a \equiv \pm b [2\pi] \\ \sin a = \sin b &\iff a \equiv b [2\pi] \text{ ou } a \equiv \pi - b [2\pi] \\ \tan a = \tan b &\iff a \equiv b [\pi] \end{aligned}$$